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USING GAME THEORY TO INCREASE STUDENTS' MOTIVATION TO LEARN MATHEMATICS

by

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USING GAME THEORY TO INCREASE STUDENTS' MOTIVATION TO LEARN MATHEMATICS

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ABSTRACT

This paper reports an attempt to teach game theory in order to increase students' motivation to learn mathematics. A course in game theory was created in order to introduce students to new mathematical content presented in a different way.

INTRODUCTION

To the question that was presented by Davis (1987):"What kinds of experience should we provide for students in order to help them to learn mathematics?" I would respond: Let us try to teach them mathematics in everyday language: by this I mean teaching mathematics using as little of the remote alienating formalistic language as we can.

In my view, in order to teach "real mathematics" we need to introduce students to as much mathematical content and as many ways of presenting that content as possible. One of the ways that is not sufficiently familiar to students is mathematical argumentation via everyday language. We must try to introduce the students to a mathematics which is essentially closer to human experience, where it will be legitimate to use everyday language.

I agree with Aumann (1985) that "what we are trying to do in science is to understand our world", and therefore "the basic aim of scientific activity remains the comprehension itself".

Thus, in teaching mathematics we must be chiefly aware of the process of understanding. Following Michener (1978),a large part of the process of understanding mathematics is connected to building and enriching the basis of knowledge. According to Courant (1967), the purpose of teaching mathematics is to bring the student to a stage of understanding mathematics as an organic whole, and this can be achieved by introducing him to diversified content of mathematics.

Having all this in mind, I decided to try to create a textbook in game theory for high-school or equivalent level. I choose game theory because "the language of game theory- coalitions, payoffs, markets, votes- suggests that it is not a branch of abstract mathematics; that it is motivated by and related to the world around us; and that it should be able to tell us something about that world" (Aumann, 1985). Yet game theory is a branch of mathematics; "the medium is mathematical model with its definitions, axioms, theorems and proofs"(Aumann, 1985).

THE COURSE IN GAME THEORY

The course in game theory is an outcome of a research for a doctorial thesis, "Teaching Game Theory in High School" (Gura, 1989).
We knew that we could not create a systematic course in game theory for high-school level due to the lack of mathematical skills. Therefore, we chose four topics that do not necessarily bear any mathematical relation to one another. The topics took into account the following criteria; their being of special interest beyond their mathematical content, not demanding specific prerequisite knowledge in mathematics, and providing general knowledge about game theory and its concerns.
We strove to write a book sufficiently varied in content and level of explication using the minimum formal apparatus as possible. Game theory as a course in school is a unique subject which is exceptional within the already existing curricula, therefore, we had to check carefully whether it is at all possible to teach game theory in high school or equivalent level and if so, at what level of explication? To what depth? In order to answer these questions it was necessary to focus on the election of topics and the actual attempt to teach these topics.
The four topics are:

Topic 1: Mathematical Matchmaking
Topic 2: Social "Justice"
Topic 3: The Shapley Value in Cooperative games
Topic 4: Financial Conflicts Discussed in the Talmud[1]

Topic 1: Mathematical Matchmaking

The Stable Marriage problem in particular and the Stable Matchmaking Problem are the underpinnings of Mathematical Matchmaking. A stable marriage, or matching, is

[1] Talmud - a 2000 years old document that forms the basis for Jewish civil, criminal and religious law

one-to-one pairing of a set of men to a set of women, contains no man and woman who would agree to leave their assigned partners in order to marry each other.
The subject of this topic draws attention to mathematical structures, assumptions, theorems, fine distinctions between types of theorems and structure of proofs and all this in everyday language and without formal apparatus. The subject of this topic provides an illustration of how mathematics may approach human experience and, therefore, also be more relevant. The topic demonstrates mathematics' power to abstract, without necessarily employing formal symbols, as well as something of the application of mathematics to society's problems.

Topic 2: Social "Justice"

The majority rule is the main way of group decision making in a democratic society. Here we are exposing the student to the voting paradox, which means that sometimes majority rule leads to a contradiction. Furthermore, at times there is no way to come to a decision after voting. We are looking for a "welfare function" that will overcome majority rule problems and will satisfy an axiomatic system that expresses our intuitive feeling for social justice. Arrow's (1951) conclusion, the impossibility theorem, proves that such a function does not exist.
The majority rule is so popular in group decision making that it is worth showing that it does not always work and we have to face the reality that there is no better solution.

Topic 3: The Shapley Value in Cooperative Games

The Shapley Value (1953) is one of the solution concepts for n-players cooperative games. Games are classified into cooperative and non-cooperative games, depending on the availability of a mechanism to enforce agreements. Some of the basic concepts of cooperative games that we present are coalitions, the characteristic function form, the transferable utility characteristic function, and superadditivity of the characteristic function. On this basis we present the Shapley Value that can be calculated for any superadditive game (n;v) in characteristic form with a finite number of players, with the additional advantage of a unique outcome that satisfies both individual and group rationality.
This topic may appear more technical than the others, but in reality contains quite elementary computations with an emphasis on concepts. This topic includes no proofs, thereby serving as an example of how one may present an interesting mathematical subject without proofs and impart general concepts relating to the capabilities and limitations of the theory; while the mathematical models are not identical with real situations, they nevertheless do explain and illuminate phenomena.

Topic 4: Financial Conflicts Discussed in the Talmud

A solution to a bankruptcy problem appears in the Babylonian Talmud (Ketubot 93a) as follows:
"There are three creditors. The debts are 100, 200 and 300. Three cases, corresponding to estates of 100, 200 and 300, are considered. The *Mishna* [2] stipulates the division shown in the table below:

Table : Mishnaic division to three creditors

Debts / Estates	100	200	300
100	33 1/3	33 1/3	33 1/3
200	50	75	75
300	50	100	150

….when E=100, the estate equals the smallest debt…equal division then makes good sense. The case of E=300 appears based on the different—and inconsistent --principle of proportional division. The figures for E=200 mysterious, but whatever they may mean, they do not fit any obvious extension of either equal or proportional division. A common rationale for all three cases is not apparent "(Aumann & Maschler, 1985, p.196).
No satisfactory solution to this *Mishna* was found for over the course of 2000 years. Using game theory, however, a solution was found. Moreover, a generalization was formulated concerning bankruptcies with an arbitrary number (n) of creditors and amount (E) of assets. The mathematical apparatus necessary for solving these problems is too complicated for high school students: Aumann and Maschler provide some simple instrument which would even have suited the Talmud sages, who obviously did not know game theory.
The subject of this topic is the most contemporary in the book. An encounter with this topic may contribute to an acquaintance with mathematics as a vital and active science.

[2] Mishna - the basic text that serves as the starting point for the discussions recorded in the Talmud

DICUSSION AND RESULTS

Our task was not to generate one more book to be added to the already existing collection of game theory texts, but to create an instructional basis for a course in game theory at the high-school or equivalent level. The fact that we did not pre-determine a definite target population required that the book be written in a manner effective for use at several levels.

We tried to reduce the mathematical prerequisites so that the book would suit the widest possible spectrum of readers and pupils. We believe that the use of this instructional volume will introduce the students to as yet unfamiliar mathematical structures and awakens his curiosity concerning mathematics as a whole, and, in particular, game theory. In general, the book may serve to enrich mathematics education by exposing students and readers to another style of presentation and to a different subject matter.

We found the book to be well suited for use as a central framework around which courses in game theory may be constructed at high and low levels of mathematics.

Our hopes for the course were realized. We found that the course in game theory contributes to the enrichment of the mathematical world-view.

As a consequence of the course, the number of student exhibiting an open minded attitude toward mathematics increased, that is, these students found themselves able to relate to mathematics not only as technical and computational, but as an expanding and developing world of its own. We found that the course in game theory contributes to a change in attitude toward mathematics in general. Subsequent to the course, many students evidenced a change in attitude toward mathematics.

Students discovered that the world of mathematics is much richer than they had previously thought.

In general, both in writing and in conversation, the students gave the impression of having enjoyed studying a new area of mathematics so very different from what they had been accustomed to in previous studies. Indeed, it would appear that the very encounter with a new sphere of mathematics, in and of itself, creates a new receptivity in the students and increase their readiness to absorb new concepts and values. Experience confirmed our supposition that through actual contact with the substance of living mathematics we might provide students with an introduction to mathematics as an organic whole. We found that the course in game theory caused a more significant turnabout among the low level students with regard to mathematics. There can be no doubt that the course in game theory was of great benefit to the high level students, but students who select mathematics as a major subject usually like it, and are, therefore, receptive to the encounter with new mathematical subject matter from the outset. By comparison, students studying mathematics at low level do not care for mathematics, and the majority study it only because it is compulsory, making the achievement of

such a turnabout in these students all the more impressive. Apparently, we had made the correct decision in not defining our target population in advance.
The fact that we were sufficiently open to any possible experience permitted teaching at different levels, and thus we achieved more interesting results.

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